Q1. Attempt any ten of the following: [10 x 2=20]

a) How many diagonals are there in a regular decagon.

b) Prove that p -> ( p v q ) is a tautology.

c) The set P( {a,b,c} ) is partially ordered with respect to the subset relation. Find a chain of length 3 in P.

d) Find the solution of recurrence relation an = 3an-1 + 1 where a0 = 1.

e) Prove that if \( \gcd(a,b)=1 \) then \( \gcd(a^2,b^2)=1 \).

f) Consider (m,3m) encoding function, where m=4. For received word 011010011111 an error will occur or not.

g) Give 2 ways to represent a graph in computer.

h) Define hamiltonian graph with example.

i) Show that any subgroup of a cyclic group is cyclic.

j) Show that if any 5 numbers from 1 to 8 are chosen, then two of them will add up to 9.

k) How many ways are there to arrange 7 –sign and 5 +sign, such that no two +sign are together.

UNIT –I

Q2. a) knight is a person who always tell truth and knave always lie. We have two people A and B such that

A says “B is a Knight”, B says “the two of us are opposite”
What are A and B?

b) Let Z be the set of all integers and R be a relation defined on Z such that for any \(a, b \in Z\), \(aRb\) if and only if \(ab \geq 0\). Is R an equivalence relation?

c) Show that a set of n elements can have \(2^n\) subsets.

Q3.a) Prove that \(|xy| = |x||y|\) is true for all real numbers \(x\) and \(y\).

b) Define function. Find the inverse of \(f(n) = 2(x - 2)^2 + 3\) for all \(x \leq 2\).

c) Find the number of integers between 1 and 100 that are divisible by any of the integer 2, 3, 5, 7.

**UNIT –II**

Q4.a) solve the difference equation

\[a_r - 5a_{r-1} + 6a_{r-2} = 2^r + r\]

for \(r \geq 2\) with the boundary conditions \(a_0 = 1\) and \(a_1 = 1\).

b) let \(L_1\) be the lattice \(D_6\) (divisor of 6) = \(\{1, 2, 3, 6\}\) and let \(L_2\) be the lattice \((P(S), \subseteq)\) where \(S = \{a, b\}\). Show that two lattices are isomorphic.

Q5.a) simplify \(y = \sum m(0, 1, 2, 3, 4, 6, 8, 9, 10, 11, 12, 14)\) using K-map.

b) Compute \(f(n)\) when \(n = 2^k\), where \(f\) satisfies the recurrence relation \(f(n) = 8f(n/2) + n^2\) with \(f(1) = 1\).

**UNIT –III**

Q6.a) Let \((G, *)\) be a group. Let \(H = \{a: a \in G\ \text{and} \ a*b = b*a \ \text{for all} \ b \in G\}\).

Show that \(H\) is a normal subgroup.

b) Is 8792002627912 a valid universal code. Explain.
c) Solve $34x=60 \pmod{98}$

Q7.a) A code $G$ contains 16 code words: $0000000$, $1111111$, $1101000$ and all its cyclic shifts, $0010111$ and all its cyclic shifts. Show that $(G, \cdot)$ is a group code. Set up the coset table to show that $G$ can correct all single transmission errors.

b) Encrypt the word ‘BOOK’ and ‘PARK’ using Caesar cipher system

$f(p) = p + 3 \pmod{26}$.

**UNIT – IV**

Q8.a) Define Eulerian graph. Prove that a non empty connected graph is Eulerian if and only if its vertices are all of even degree.

b) Differentiate between

i. Graph and Tree
ii. Sub graphs and isomorphic graph
iii. Connected and complete graph

Q9.a) Prove that a planar graph $G$ is 5 colorable.

b) Explain inorder, preorder and postorder tree traversals with the help of an example.